Lecture Notes: Introduction to FEA/FEM

FEA/FEM:

- Goes back to ancient mathematicians. Archimedes used the basic technique with a 96-sided polygon to approximate pi to 30 decimal places! (circumference = $2\pi r$)
- Used in structural design as a numerical approximation method to solve complex structural problems
- Analysis methods:
    - Linear (E = constant)
    - Nonlinear (due to nonlinearities in material, geometry (contact, etc.), BC’s, etc.)
    - Dynamic (ballistic, impact, transient, harmonic/vibration)
    - Heat
    - Fluid

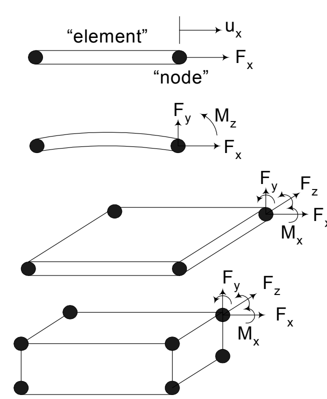
- A structure is divided up into a number of discrete sub-structures or “elements”

    - Each “element” is connected to the adjacent elements at “nodes”, where forces and moments (loads) are applied and displacements (translations and rotations) are determined.

    - For the total structure, which is made up of many “elements”, the laws of solid mechanics are applied:
        - Equilibrium of forces and moments
            - Within each element and between elements
        - Compatibility of displacements (translations and rotations)
            - Within each element and between elements (continuity)
        - Laws of material behavior (stress-strain laws)
            - Within each element

    - The mechanical behavior of each element is modeled using equations from solid mechanics. Examples:
        - Truss element (rod or bar) (1-D)
            - 2 nodes/element, 1 DOF/node
        - Beam element (1-D, 2-D)
            - 2 nodes/element, 2-3 DOF/node
        - Plate/shell element (2-D, 3-D)
            - 3-4 nodes/element (triangular, quad), 4-6 DOF/node
        - Solid element (brick) (3-D)
            - 8 nodes/element, 3-6 DOF/node

        - Plus many variations on these basic ones! (element libraries)



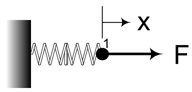
- Once the structure is modeled using these discrete elements, interconnected at the nodes, then the loads are applied at each node (nodal forces), and then the displacements and stresses are determined using matrix methods.

So, FEA is a numerical <u>approximation</u> of a complex structure by breaking it up into small (discrete) parts, whose mechanical behavior can be better modeled and predicted than the entire structure as a whole.

Note that the model can be made more accurate by making the element sizes smaller, but it is still only as good as the simplified mechanics equations used (continuum mechanics)!

HOW: Elements are modeled as spring-like objects

- One spring



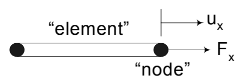
$$F = k \cdot x$$

Applied force

"stiffness" of spring (force/displacent) (lb/in)

displacement of point 1

- FEA w/ one DOF



$$F = k \cdot u$$

Applied force in each DOF (say x-direction)

stiffness of element in each DOF (say x-direction)

displacement of each DOF (say x-direction)

Note for a truss (1-D) element: $k = \dfrac{EA}{L}$

- FEA for multiple (many) elements

$$\{F\} = [K] \cdot \{U\}$$

Array of applied forces (one for each DOF)

Matrix of stiffnesses (DOF x DOF)

Array of displacements (one for each DOF)

$\{F\}$ is "known" (loads)
$[K]$ is "known" (geometry, material properties…elements)
$\{U\}$ is to be determined (displacements)
This can be solved mathematically using a <u>matrix inversion</u> method
$\{F\} = [K] \cdot \{U\} \quad \rightarrow \quad \{U\} = [K]^{-1}\{F\}$

Once the displacements $\{U\}$ are known, then strains and stresses can be determined:

$\varepsilon = \dfrac{\Delta u}{L}$ (1-D …more complicated for 2-D and 3-D strains)
$\sigma = E \cdot \varepsilon$
and $FOS = \dfrac{\sigma_y}{\sigma}$

→ Much more on mathematics later !
→ See handout (articles on FEA)
→ See Powerpoint (website)